Box 2: Snow-ball effects and stock flow adjustments

The outstanding level of public debt evolves from one year to the next depending on the yearly fiscal turnout, according to the simplest fiscal accounting framework. When a country runs a fiscal deficit (that is when government's total expenditure is higher than its total revenue), the outstanding level of public debt increases, and when the country runs a fiscal surplus (that is when government's total revenue is higher than its total expenditure) the outstanding level of public debt diminishes. This relationship can be expressed in alternative ways, as in equations [1] to [5], with subscript (t) indicating the current time period, subscript (t-1) indicating the previous year, and (r) referring to the interest rate (assumed to be constant for simplicity). In particular, equation [5] evidences that in order to maintain the outstanding level of public debt constant the country would need to run a primary surplus (a positive primary balance) sufficient to match the yearly debt interest payments.

[1] $\text{Debt}_t - \text{Debt}_{t-1} = -\text{Fiscal Balance}_t$

[2] $\text{Debt}_t - \text{Debt}_{t-1} = \text{Total Expenditure}_t - \text{Revenue}_t$

[3] $\text{Debt}_t - \text{Debt}_{t-1} = (\text{Primary Expenditure}_t + \text{Interest Payments}_t) - \text{Revenue}_t$

[4] $\text{Debt}_t - \text{Debt}_{t-1} = (\text{Primary Expenditure}_t - \text{Revenue}_t) + (r * \text{Debt}_{t-1})$

[5] $\text{Debt}_t - \text{Debt}_{t-1} = -\text{Primary Balance}_t + (r * \text{Debt}_{t-1})$

To allow cross-country comparability, as well as to better evaluate the outstanding level of debt in relation to the country's repayment capabilities (particularly in the case of a growing economy) it is helpful to express fiscal variables in terms of ratios to GDP. Dividing each of the fiscal variables in equation [5] by nominal GDP (labelled as Y), yields the following equation:

[6] $(\text{Debt}_t / Y_t) - (\text{Debt}_{t-1} / Y_t) = (-\text{Primary Balance}_t / Y_t) + [(r * \text{Debt}_{t-1}) / Y_t]$

To better identify the implications of equation [6] it is also useful to multiply and divide the variables related to period (t-1) with Y_{t-1} (that is the previous year's nominal GDP level), as this transformation allows for a richer analysis of the public debt dynamics. Lower case letters in equation [7] indicate the variables expressed as a ratio to the same period GDP, with (d) and (pb) representing respectively the debt-to-GDP ratio and the primary balance-to GDP ratio, while (g) represents the growth rate of nominal GDP from period (t-1) to period (t), also assumed to be constant for simplicity.

[7] $d_t - [d_{t-1} / (1+g)] = [-pb_t] + [(r * d_{t-1}) / (1+g)]$

Re-arranging the terms in equation [7] yields equation [8].

[8] $d_t - [d_{t-1} (1+ r) / (1+g)] = -pb_t$

For small value for (r) and (g), equation [8] can also be approximated as follows:

[9] $d_t - [d_{t-1} (1 + r - g)] = -pb_t$

The so called '**snow-ball effect**' is defined as the interaction term between the public debt-to-GDP ratio for the previous year and the difference between the interest rate on public debt and the nominal GDP growth rate, referred to as the 'interest-growth differential'. Expressed differently, the so called "snow-ball effect" consists of the upward push on the outstanding debt as a result of the debt servicing costs, net of the downward effect on the debt ratio generated as a result of real economic growth and inflation (which push up the GDP denominator). The 'snow-ball effect can thus be positive when the interest rate is higher than the GDP growth rate and negative when the interest rate is lower than the GDP growth rate.

Therefore, whenever the interest rate is higher than the nominal GDP growth rate (which is the more common type of situation under normal economic conditions) there is need for a primary fiscal surplus, to maintain the debt-to-GDP ratio stable. On the other hand, when the nominal GDP growth rate is higher than the interest rate on public debt, the debt ratio would fall when there is a primary balance (when total revenue matches the primary expenditure, which is equivalent to total expenditure net of interest payments).

In this respect, it is pertinent to note that nominal growth reflects the combined effect of real growth and inflationary pressure, and as a result, within this framework, both real GDP growth and inflation help contain the trajectory in the debt ratio. Consequently, against a background of rather subdued inflation, it is important the real economic growth remains vibrant, to maintain nominal GDP growth elevated.

Moving away from the simplest accounting framework, it is possible to identify other factors which impact the evolution of the debt-to-GDP ratio, beyond the primary balance and the 'snow-ball effect'. These other factors are collectively referred to as '**stock-flow adjustments**' and give rise to a situation wherein, the annual change in gross debt does not equal the annual fiscal deficit. These factors include:

- Valuation effects, through the impact of say exchange rate changes on foreign currency denominated debt;
- Time of recording effects, since deficits are often measured in accrual terms while debt is a cash concept; and
- Below the-line operations, such as privatisation revenues and transactions in financial assets, such as the acquisition or disposal of shares.

Equation [9] can thus be expanded in order to incorporate the stock-flow adjustments as per cent of GDP, represented as (sf) in equation [10], which can push up or down the debt-to-GDP ratio.

[10] $d_t - [d_{t-1}(1 + r - g)] = - pb_t + sf_t$

As can be seen from equation [10] positive stock-flow adjustments push up the debt ratio whereas negative stock-flow adjustments lower the debt ratio.